

Department of Mathematics
Faculty of Mathematics & Computer Science
M.Sc. (Applied Mathematics), 4th Semester

Course Code	AM 403 (a)
Course Title	Measure and Integration
Course Credits	04

Course objectives:

Minimum pre-requisites:

Basic undergraduate knowledge of set theory and Riemann integration will be useful.

Course structure:

The contents of this course form an essential setting for application courses, e.g., Fourier Analysis, Sobolev spaces, weak solutions of PDE etc.

Countable sets, uncountable sets, Cardinal numbers of the set of natural numbers, the set of real numbers and the set of functions, Order relation between these cardinal numbers.

Algebra of sets, the extended real numbers, Borel sets, Countably additive measures.

Lebesgue measure: Outer measure, Measurable sets and Lebesgue measure, Non-measurable sets, Measurable functions, Littlewood's three principles, Egoroff's Theorem.

Lebesgue Integral : Simple functions, integral of a simple function, Lebesgue integral of a bounded measurable function over a set of finite measure, Comparison of Riemann and Lebesgue integrals, Theorem of bounded convergence, The integral of a non-negative measurable function, Fatou's Lemma,, Monotone Convergence Theorem, the general Lebesgue

integral, Lebesgue's theorem of dominated convergence. Convergence in measure.

Differentiation and Integration: Differentiation of monotone functions, Functions of bounded variation, Differentiation of an integral, Absolutely continuous functions, Theorem that a function is an indefinite integral if and only if it is absolutely continuous.

The L^p -spaces, Hölder and Minkowski inequalities, Completeness of L^p -spaces.

Introduction to General Measure and Integration: Measure spaces, Measurable functions, Integration.

Reading suggestions:

- G. de Barra, *Introduction to Measure Theory*, Van Nostrand Reinhold Company, New York, 1974.
- R.G. Bartle, *The Elements of Integration and Lebesgue Measure*, John Wiley & Sons, Inc., New York, 1995.
- J. N. McDonald and N. A. Weiss, *A Course in Real Analysis*, Academic Press, New York, 1999.
- H.L. Royden, *Real Analysis*, (3rd ed.), The Macmillian Company, New York, 1988.

Evaluation and weightage:

- **Mid-Term Examination (20%):** During the middle of the session, there will be a written examination.
- **Assignments (20%):** In all 4 assignments will be given, one in each month.
- **Quiz/Presentations (20%):** As per the lecture schedule, Quiz session will be organized and students will be asked to make presentations. The topics will be assigned during the lectures.
- **Term-End Examination (40%):** At the end of the session, there will be another written examination.

