Department of Mathematics
Faculty of Mathematics & Computer Science
PhD, Mathematics

Course Code	AM 601
Course Title	Weighted Norm Inequalities and Integral Operators on Function Spaces
Course Credits	04

Course objectives:

Weighted integral inequalities of Hardy type are important ingredient to study imbeddings of Sobolev spaces into Lebesgue spaces which in turn play significant role in obtaining weak solutions of various partial differential equations. The weight classes Ap and Bp are used to characterize the boundedness of several integral operators that have connections in Harmonic Analysis, Wavelet Theory etc. This course would serve as a stepping stone for those who wish to pursue research in Harmonic Analysis, Fourier & Wavelet Theory, Sobolev Spaces, weak solutions of initial value & boundary value problems and related areas.

Minimum Pre-requisite:

A first course both in Functional Analysis and Measure theory will be useful.

Course structure:

Hardy type inequalities: One dimensional inequalities (involving the Hardy operator H as well as its adjoint H^*) with Muckenhoupt conditions as well as Persson-Stepanov conditions, Method of differential equations, Higher order inequalities involving operators defined over balls, Higher dimensional inequalities, Norm inequalities involving fractional integral operator of Riemann Liouville type, Extreme cases, Geometric mean operator as a limiting case of Hardy operator, Hardy-Steklov operator, Generalized Hardy operator involving Oinarov kernels.

Integral inequalities for monotone functions: Duality Principle of Sawyer, Hardy's inequalities on cone of monotone functions.

Classes of weights: A_p-class of weights and its properties, Maximal operator and its L^p-boundedness, B_p-class of weights and its properties, B_p*-class of weights and its properties, L^p_{dec}-boundedness of Hardy averaging operator.

Orlicz spaces: Young's function, Jensen's inequality, Complementary Young's function, Δ_2-condition, Orlicz spaces, Hölder's inequality, Luxemburg norm, Completeness, Convergence, Comparison of Orlicz spaces, Hardy-type inequalities in Orlicz spaces.

Reading suggestions:

- **R. E. Castillo and H. Rafeiro**, *An Introductory Course in Lebesgue Spaces,* CMS Books in Mathematics, Springer, Switzerland, 2016.
- **J. Duoandikoetxea**, *Fourier Analysis,* Vol. 29, Graduate Texts in Mathematics, Amer. Math. Soc., RI, 2001.
- **A. Kufner, L. Maligranda and L.E. Persson,** The Hardy Inequality. About Its History and Some Related Results, Vydavatelsky Servis Publishing House, Pilsen, 2007.
- **A. Kufner, L.E. Persson and N. Samko,** *Weighted Inequalities of Hardy Type,* Second Edition, World Scientific New Jersey, 2017.
- **C.J. Neugebauer**, *Weighted norm inequalities for averaging operators of monotone functions,* Pub. Mat., 35 (1991), 429-447.
- **C.J. Neugebauer**, *Some classical operators on Lorentz space,* Forum Math. 4 (1992), 135-146.
- **B. Opic and A. Kufner**, *Hardy-Type Inequalities,* Pitman Research Notes in Mathematics Series, Longman Scientific and Technical Harlow, 211, 1990.
- **L. Pick, A. Kufner, O. John and S. Fucik**, *Function Spaces 1,* Volume 14/1 in the series De Gruyter Series in Nonlinear Analysis and Applications, 2013.

Evaluation and weightage:

- Mid-Term Evaluation Examination: 20%
- Presentations: 40%
- End-Term Evaluation: 40%