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Endogenous Inequality and Mobility in a Neoclassical Growth Model with Hand-to-Mouth Households

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Abstract

This paper develops a neoclassical growth model with endogenous socio-economic mobility between optimizing 'capitalist' households and poor 'hand-to-mouth' households, where the latter are unable to participate in capital accumulation due to a heterogeneous minimum consumption floor. We provide microeconomic foundations for persistent social classes and endogenize mobility between social classes during the process of economic growth. The model demonstrates that poverty and inequality leads to inefficiency in the competitive market equilibrium, where the welfare theorems fail. Redistribution measures like those based on a tax on capital income improves welfare. Further, in the presence of historically disadvantaged 'resilient' hand-to-mouth households, affirmative action measures improve welfare. The paper contributes to the existing literature by integrating class dynamics into growth theory in a simple and analytically tractable framework.

JEL Classification: E13; E21; D15; D31; D63
Keywords: Neoclassical growth, inequality; heterogeneous agents, hand-to-mouth agents, capital income tax, welfare theorems, affirmative action.

1 Introduction

The main objective of this paper is to develop a theory of inequality as a macroeconomic variable, and its relation with important macroeconomic processes. We attempt to describe this relation with the simplest possible story which is consistent with the available empirical evidence. We outline below one such possible story.

Consider the standard textbook version of a neoclassical growth model (e.g. Bénassy 2011), where the economic growth is driven by the process of capital accumulation. The latter is determined by an intertemporal optimization exercise run by representative households with infinite horizon. In a world characterized by inequality and poverty, however, there are households which live hand-to-mouth, struggling to meet their essential consumption requirements (see, for instance, Deaton 1991; Banerjee and Duflo 2007). Hence, they are not in a position to run the intertemporal optimization exercise to participate in capital accumulation till their essential consumption requirements are met. This leads to suboptimal growth. Further, as the economy moves through this suboptimal growth path, various economic processes change the distribution of income (or the proportion of hand-to-mouth households), which in turn affects the growth trajectory. We wish to describe this two-way relationship through the simplest tractable macroeconomic framework. This leads us to choose the benchmark neoclassical growth model as our starting point and incorporate inequality as a macroeconomic variable within this framework.

The paper contributes to the growing literature on macroeconomic models with heterogeneous agents. In a departure from this literature, however, we endogenize socio-economic inequality in a neoclassical growth model. We incorporate heterogeneity within a population divided into socio-economic classes, with mobility between the various social classes. As we elaborate in rest of the paper, this mobility between social classes allows us to include inequality as a dynamic macroeconomic variable (as opposed to a time-invariant or a stable parameter, as is the case with at least a section of the existing literature). We examine the various dynamical possibilities arising out of the interaction of social mobility with business cycles.

Our study is closely connected with at least three distinct lines of literature. We discuss each of these briefly in the following paragraphs.

Firstly, as we pointed above, there is a large existing literature consisting of theoretical models with heterogeneous agents in incomplete markets (mainly due to the presence of an uninsurable idiosyncratic risk), starting from the early work of Aiyagari (1994), Chatterjee (1994), Aiyagari (1995), Krusell and Smith (1996), Constantinides and Duffie (1996), and Krusell, Smith, and Anthony (1998). Heathcote, Storesletten, and Violante (2009) contains a comprehensive review of this literature. Some recent contributions in this line of literature examine the relationship between inequality and economic growth through various channels like endogenous labor supply (Turnovsky and García-Peñalosa 2008; Bosi and Seegmuller 2010; García-Peñalosa and Turnovsky 2015), shifts in productivity (Lee 2023), effect on physical and human capital accumulation (Sugimoto 2006), heterogeneity in human capital (Yuki 2016; Mu, Yan, and Wu 2023), heterogeneity in altruism (Borissov, Bosi, Ha-Huy, and Pakhnin 2025) or through the political economy considerations and its effect through minimum wage legislations (Tamai 2009). Another related line of literature examines the relationship between inequality and economic growth through shifts in the bias of technical change (see, for instance, Acemoglu 1998; Acemoglu 2002). However, the focus of our study is to examine the two-way relationship between inequality and economic growth in isolation from the other contributing factors. In particular we are interested in how the fundamental economic processes which drives economic growth and inequality are related with each other. We feel that the dynamic general equilibrium models with limited asset market participation (LAMP) are more appropriate for this purpose. Bilbiie (2008) is an early example of this line of literature, where some of the households are 'hand-to-mouth' who consume what they earn and do not participate in the asset markets to smoothen consumption.[1] Subsequently, this framework has been used in contributions in several directions (e.g. Motta and Tirelli 2012; Areosa and Areosa 2016; Piergallini 2017; Maebayashi and Tanaka 2022).

However, our current study is the closest to Gomes (2021), which includes heterogeneity of agents in a neoclassical growth model in a framework similar to Bilbiie (2008). Unlike the New Keynesian model of Bilbiie (2008) which includes imperfect competition, Calvo pricing and money, the model in Gomes (2021) represents a more barebone neoclassical (Ramsey-Cass-Koopmans) growth model. We choose this relatively simpler version of the neoclassical growth model from Gomes (2021), rather than the version used by Bilbiie (2008) to isolate the effect of heterogeneity and social mobility in this class of

[1] This classification of households based on whether they participate in the asset markets has also been used earlier in the Post Keynesian literature (see, for instance, Kalecki 1971). In this line of literature, 'workers' consume what they earn and 'capitalists' save a part of their income. However, the consumption and saving decisions are not made using an optimization exercise.

models. Like Gomes (2021), we include poor 'hand-to-mouth' consumers in the neoclassical growth models who are not in a position to save. Hence, unlike optimal consumers (or 'capitalists') who makes decisions on consumption based on an optimization exercise, the poor hand-to-mouth consumers consume their entire income without running any optimization exercise.[2] However, in much of the literature on general equilibrium with limited asset market participation, the classification of households into the social classes is ad-hoc and exogenously fixed. We contribute to this literature by attempting to address both these concerns; the former by providing an economic basis for this classification and the latter by explicitly allowing for endogenously determined social mobility between these classes.

There is a substantial empirical literature which supports this particular approach we adopt for our theoretical modeling. This is the second line of literature which is connected with our study. To begin with, we point to a general lack of empirical evidence of consumption smoothening. In the standard textbook version of the neoclassical growth model, optimal agents smoothen intertemporal consumption over infinite horizon. However, empirical evidence (see, for instance, Campbell and Mankiw 1989) shows that a substantial fraction of the population in major OECD countries do not hold any substantial assets, and for whom, consumption is sensitive to current income. On similar lines, Dynan, Skinner, and Zeldes (2004) finds that rich save a much higher fraction of their lifetime incomes than the poor, and this difference cannot be explained by the differences in preferences. More recently, Krueger, Perri, Pistaferri, and Violante (2010) provides, among others, an empirical justification for taking heterogeneity in macroeconomics seriously.

Finally, our paper is also connected with the literature on macroeconomic impacts of measures to address inequality like income and wealth taxation. There is a long line of literature in this direction; Aiyagari (1995) is one of the early contributions, more recent contributions include, among others, Laroque (2011), Guvenen, Kambourov, Kuruscu, Ocampo, and Chen (2023), and Piketty, Saez, and Zucman (2023). We examine how such redistribution measures are transmitted through the macroeconomic channels of a simple neoclassical growth model. However, a serious assessment of such redistribution measures or a comparison of alternative measures (like income and wealth tax) from policy perspective would require setting up a more realistic and complete macroeconomic

[2]We should clarify here that we do not consider wealthy hand-to-mouth households in our model, in the sense discussed, among others, by Kaplan, Violante, and Weidner (2014). Households of this type, unlike poor households, are temporarily hand-to-mouth for certain part of their life cycle due to liquidity constraints. One of the models in Gomes (2021) incorporates the wealthy hand-to-mouth.

model. Since we emphasize tractability of thhe model, we refrain from a more complete analysis of wealth taxation in the current study.

In light of the existing literature we discussed above, the main objective of the current study is to incorporate inequality as an endogenously determined dynamic macroeconomic variable in a benchmark neoclassical growth model. In other words, we do not seek to offer a complete realistic model here which incorporates factors like human capital or technology. We feel that while more complex models, incorporating factors like human capital or technology might be closer to the real world, a simpler framework might be in a better position to highlight the role played by inequality which the more complex models might miss. This is the main reason for us to pick the standard textbook version of a Ramsey-Cass-Koopmans model (Ramsey 1928; Cass 1965; Koopmans 1965) of neoclassical growth as our starting point and incorporate inequality as a dynamic macroeconomic variable in this framework.

We begin in section 2 with a brief overview of some empirical stylized facts on inequality. We use these stylized facts as a guide to construct a benchmark neoclassical growth model with inequality in section 3. We examine the dynamic properties of this model. Then in section 4, we examine the efficiency of the competitive markets in the presence of inequality. We follow this in section 5 with an analysis of the efficiency of a tax on capital income. In section 6, we incorporate the presence of historically disadvantaged groups of resilient hand-to-mouth households and investigate the efficiency of affirmative action policies. Finally, in section 7 we conclude.

2 Dynamics of inequality: some empirical stylized facts

In this section, we use the data on income inequality and wealth inequality from World Inequality Database to draw our attention to certain empirical stylized facts. In particular, we use the indicator 'Bottom 50% share' for both income and wealth inequality as the main indicator of inequality. The reason for choosing the 'bottom 50% share' rather than the share of top 10% or 1% lies in the fact that the bottom share seems to have a closer relation with the indicator of inequality in our theoretical model which we develop in the following sections. In the theoretical model, we use the 'hand-to-mouth' households who do not have access to the asset markets as the indicator of inequality. We are of the view that the bottom 50% share in income and wealth would be a better proxy for this indicator than any of the top percentiles.

Further, we are interested in modeling the indicator of inequality as a macroeconomic variable which changes over time. Hence we look at the time-series properties of its empirical counterpart, which will provide us with some clue about how to model this variable in our theoretical model. For our sample, we choose three high income countries (USA, Great Britain and France), three upper middle income countries (Brazil, China and South Africa) and one lower middle income country (India). The main criterion for this choice has been the availability of data; however, since the empirical exercise which follows is meant to be indicative in support of some of the modelling choice we make in the following sections, we do not attempt to claim that this is representative of the inequality trends in the entire world.

Table 1: Summary statistics of bottom 50% share of income in the sample

	Brazil	China	France	Great Britain	India	USA	South Africa
Mean	0.09	0.18	0.21	0.20	0.20	0.17	0.12
Median	0.09	0.17	0.21	0.20	0.21	0.16	0.12
Maximum	0.11	0.25	0.23	0.24	0.24	0.21	0.16
Minimum	0.08	0.14	0.18	0.18	0.14	0.13	0.06
Std. Dev.	0.01	0.04	0.01	0.01	0.03	0.03	0.04
Observations	45	47	62	45	62	62	45

A brief summary statistics of the data on bottom 50% share in national income is provided in table 1. We find that the mean share of bottom 50% in national income is the highest (i.e. income inequality is the lowest) at 21% in France and the lowest (i.e. income inequality is the highest) at 9% in Brazil among the sample countries. Overall, the highest share of bottom 50% in national income of any country at any time in the sample is 24% for Great Britain and India. Similarly, the lowest share of bottom 50% in national income of any country at any time in the sample is 6% in South Africa. The standard deviation is the highest in case of South Africa and China (representing substantial shifts in income distribution) and the lowest in case of Brazil and France. Overall, the data seems to suggest variability in income inequality, but the standard deviations are is not very large, which might possibility be due to a high persistence in the data.

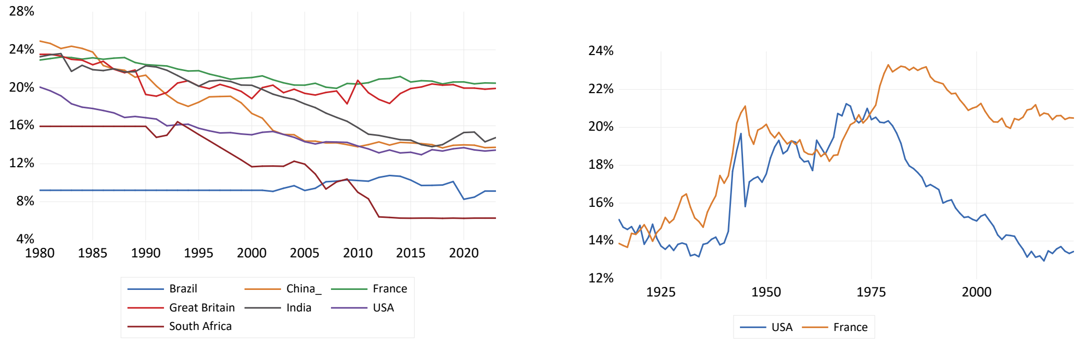
A brief summary statistics of the data on bottom 50% share in national wealth is provided in table 2. We find that the mean share of bottom 50% in national wealth is the

Table 2: Summary statistics of bottom 50% share of wealth in the sample

	Brazil	China	France	Great Britain	India	USA	South Africa
Mean	0.00	0.09	0.07	0.05	0.07	0.01	-0.03
Median	0.00	0.07	0.06	0.05	0.08	0.01	-0.02
Maximum	0.00	0.16	0.10	0.06	0.12	0.02	0.00
Minimum	0.00	0.06	0.04	0.04	0.05	0.00	-0.08
Std. Dev.	0.00	0.03	0.02	0.01	0.02	0.00	0.02
Observations	29	29	61	29	32	62	31

highest (i.e. wealth inequality is the lowest) at 9% in China and the lowest (i.e. wealth inequality is the highest) at -0.03% in South Africa among the sample countries. Overall, the highest share of bottom 50% in national wealth of any country at any time in the sample is 16% for China. Similarly, the lowest share of bottom 50% in national wealth of any country at any time in the sample is -0.08% in South Africa. The standard deviation is the highest in case of China (representing substantial shifts in income distribution) and the lowest in case of Brazil and USA. Overall, the data seems to suggest smaller variations in wealth inequality relative to income inequality, suggesting that wealth inequality tends to remain more stable. Also, in general wealth inequality is much higher than income inequality. The data seems to indicate the possibility of a substantial fraction of population not holding any wealth during certain periods within the sample. It also suggests that this fraction changes over time, due to some agents shifting from one group to another.

We plot the income inequality and wealth inequality in selected sample countries in figure 1 and 2 respectively. The figures seem to suggest the presence of both a trend as well as fluctuations around it. Income inequality figures suggest a general worsening of income distribution since 1980s in most countries. For USA and France, from 1b we find the poor 50% improved their conditions till around the onset of 1980s, after which their conditions, like other countries, worsened. The wealth inequality plots, on the other hand, show a greater degree of stability and persistence. In France, wealth distribution trends are similar to those of income distribution, with an improvement till 1980s, and a sharp worsening thereafter. For USA, wealth inequality is more stable with much smaller fluctuations.

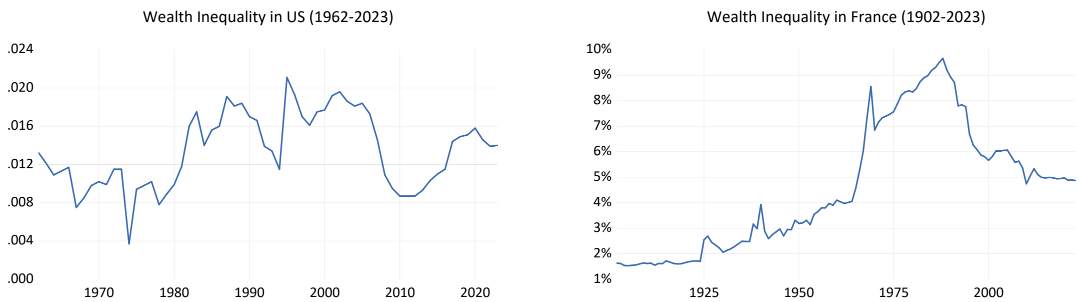


(a) Income inequality time-series in 3 HICs and 4 MICs: 1980-2023

(b) Income inequality time-series in USA and France: 1915-2023

Figure 1: Share of bottom 50% population in national income

Source: Based on the data from World Inequality Database

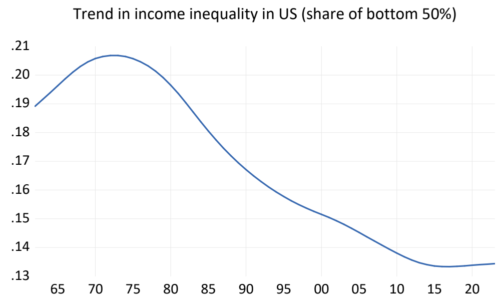


(a) Wealth inequality time-series in USA: 1962-2023

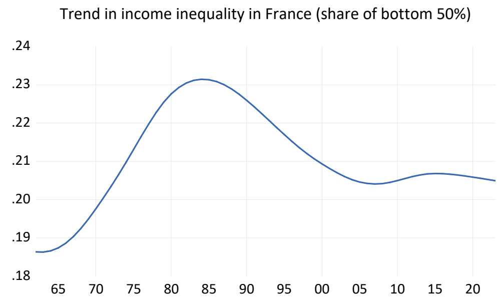
(b) Wealth inequality time-series in France: 1902-2023

Figure 2: Share of bottom 50% population in national wealth

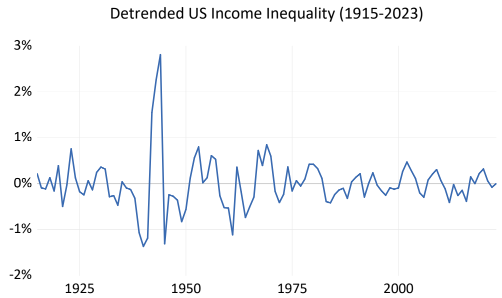
Source: Based on the data from World Inequality Database



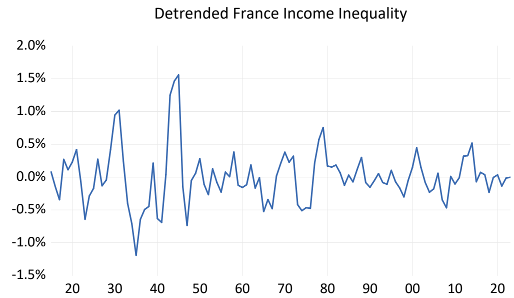
(a) Income inequality trend in USA: 1915-2023



(b) Income inequality trend in France: 1915-2023



(c) Detrended income inequality in USA: 1915-2023



(d) Detrended income inequality in France: 1915-2023

Figure 3: Trends and cycles in the share of bottom 50% population in national income using H-P filter

Source: Author's calculation from World Inequality Database

The figures on income and wealth inequality in figure 1 and 2 also suggest that the data on inequality has both a trend component as well as fluctuations or a cyclical component around the trend. We examine these components separately in figure 3 for the share of bottom 50% in national income in USA and France, using the Hodrick-Prescott filter (Hodrick and Prescott 1997). Figure 3a and 3b shows the trend component, whereas figure 3c and 3c shows the data after detrending. It is evident from figure 3 that the data on the share of bottom 50% in national income has a distinct trend and cyclical component.

Having established that there are distinct trend and cyclical components in our inequality data, next, we examine the trend component more closely by investigating the unit root properties. In order to achieve this, we run the standard unit root tests, i.e. the Augmented Dickey Fuller (ADF) test (Dickey and Fuller 1981), the Phillips-Perron (PP) test (Phillips and Perron 1988) and the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test (Kwiatkowski, Phillips, Schmidt, and Shin 1992) on the data on the share of bottom 50% in national income and national wealth of USA and France. The results from these tests for the model with intercept and trend are shown in table 3.

Table 3: Unit root properties of income inequality in US and France

	ADF		PP		KPSS	
	Level	First diff.	Level	First diff.	Level	First diff.
US Income Inequality	-1.19	-7.82	-1.04	-10.96	0.27	0.10
	(0.91)	(0.00)	(0.93)	(0.00)		
France Income Inequality	-1.65	-8.11	-1.40	-8.01	0.23	0.05
	(0.76)	(0.00)	(0.86)	(0.00)		
US Wealth Inequality	-2.57	-8.70	-2.54	-8.91	0.16	0.08
	(0.29)	(0.00)	(0.31)	(0.00)		
France Wealth Inequality	-2.86	-3.54	-3.64	-6.50	0.20	0.18
	(0.18)	(0.04)	(0.03)	(0.00)		

Augmented Dickey-Fuller test. Numbers in brackets represent p-values. Test equation includes both trend and intercept. Lag length is chosen automatically using Schwarz information criterion.
Phillips-Perron test. Numbers in brackets represent p-values. Test equation includes both trend and intercept. Bandwidth is chosen automatically using Newey-West method.
Kwiatkowski-Phillips-Schmidt-Shin test. Test equation includes both trend and intercept. Bandwidth is chosen automatically using Newey-West method. The critical values at 1%, 5% and 10% levels are 0.22, 0.15 and 0.12 respectively.

The results from the unit root tests in table 3 indicate that we are unable to reject the presence of unit root in levels but able to reject it in first difference for both income and wealth inequality in USA and France from the ADF tests. The PP tests, on the other hand, show that we are unable to reject the presence of unit roots for all the variables except wealth inequality in France, but able to reject it in first difference. The KPSS tests show that we are able to reject the null hypothesis of stationarity at levels for all the variables at 5%, and unable to reject it in first difference for all variables except wealth inequality in France. Overall, we can conclude that all the variables examined here are $I(1)$ except the wealth inequality in France, where the results are somewhat unambiguous.

More generally, the above discussion points to certain stylized facts about the nature of inequality as measured by the share of the bottom 50% population in national income and national wealth. First of all, the empirical data clearly shows that inequality in distribution of income and wealth is better modelled as a macroeconomic variable which varies over time, rather than a time-invariant parameter. Secondly, the variation in the indicator of inequality has both a trend and a cyclical component. Thirdly, preliminary investigations seem to suggest the presence of both a deterministic as well as a stochastic trend. Overall, there seems to be a substantial persistence in the data on inequality, as measured using the share of bottom 50% in national income and national wealth.

However, we hasten to add a bit of caution that the exercise in this section is meant to merely provide some preliminary and indicative guide as to how to model inequality in the context of a macroeconomic model, It is not meant to be a rigorous econometric exercise, which we leave as a future possible extension of this project. We next turn to the theoretical model, where we use some of the information from above as an indicative guide as to how to proceed our model.

3 Neoclassical growth with social mobility

3.1 The environment

Consider a neoclassical growth model in continuous time. The economy consists of identical profit maximizing perfectly competitive firms and a continuum of households, each indexed by i such that $i \in [0, 1]$. Households are infinitely lived. Each household inelastically supplies one unit of labor. Hence, total population, N is equal to the total supply of labor, L. Laborforce grows at the rate n, i.e.

$$\frac{\dot{L}}{L} \equiv n \tag{1}$$

We, however, depart from the standard neoclassical model by introducing heterogeneity of households due to limited asset market participation (LAMP, Bilbiie 2008). In order to start participating in the accumulation of capital, each household i must start by saving a part of its wage income after meeting its essential consumption requirements in the form of a *minimum consumption floor*, $c_{\min,i}$ (see, for instance, Shefrin and Thaler 1988; Deaton 1991), i.e.

$$w \geq c_{\min,i}; \quad c_{\min,i} \in (0, \tilde{c}] \ \forall\, i \in [0, 1] \tag{2}$$

where w is the wage rate and $\tilde{c}$ is an upper bound to minimum consumption floor. The minimum consumption floor, $c_{\min,i}$ differs across households (Chetty and Szeidl 2007). The main reason for such a floor might be basic survival (see, for instance, Banerjee and Duflo 2007), variable health expenditure (which is often subject to health shocks) (e.g. De Nardi, French, Jones, and McCauley 2016), prior commitments and buffer stocks (Carroll, Hall, and Zeldes 1992). These commitments vary not just across households in an economy, but also for the same household across time. For simplicity, we consider $c_{\min,i}$ to be i.i.d. with a uniform distribution between 0 and $\tilde{c}$:

$$c_{\min,i} \sim U(0, \tilde{c}) \tag{3}$$

In each small interval of time, δt, each hand-to-mouth household i draws a new $c_{\min,i}$ from a uniform distribution. Following this, the households which do not satisfy condition (2) are not able to smoothen consumption by participating in capital accumulation and instead live hand-to-mouth, consuming all their wages. Other than the heterogeneous minimum consumption floor, households are identical in all other respects.[3] Apart from existing hand-to-mouth households who are unable to escape poverty from (2), some additional capitalist households also join the rank of hand-to-mouth households in each infinitesimally small time period, δt from being hit by idiosyncratic shocks. We elaborate on the transition between different types of households in section 3.5.

The above formulation leads us to an economy similar to the one discussed by Bilbiie (2008), Bilbiie (2020), and Gomes (2021), where the economy consists of two types of households:

1. Poor hand-to-mouth households, who consume whatever they earn. There are L^h such households.
2. Optimal consumers or 'capitalists', who choose consumption path by optimizing a CRRA utility function. There are L^o such households.

A fraction, λ, $\lambda \in [0, 1)$ of the households are poor 'hand-to-mouth', i.e.

$$\lambda \equiv \frac{L^h}{L}$$
$$\text{where } L \equiv L^h + L^o$$

[3]We should note here that a full-fledged heterogeneous agent model should also take into account heterogeneity in holding of capital stock as well as wage rates. However, in order to isolate the effect of heterogeneity in $c_{\min,i}$ and to maintain the tractability of the model, we do not introduce any further heterogeneity in the model and leave the above as possible future extensions of our study.

$$\therefore \frac{L^o}{L} = \frac{L - L^h}{L} = 1 - \frac{L^h}{L}$$
$$\Rightarrow L^o = (1 - \lambda) L \tag{4}$$

Hand-to-mouth households do not save, and therefore do not contribute to the accumulation of capital. Wages are their only source of income. The optimal 'capitalist' households, on the other hand, earn their income from both wages and profits. At each time period, t, the total stock of physical capital, $K(t) \geq 0$, is concentrated in the hands of these optimal 'capitalist' households who conduct optimal plans.

3.2 Representative firm's problem of profit maximization

Let the aggregate output in the economy be given by Y. Output is produced using a linearly homogeneous neoclassical production function with constant returns to scale, i.e. $Y = F(L, K)$ with $F_L, F_K > 0$, $F_{LL}, F_{KK} < 0$, along with the Inada conditions. For the sake of specificity, we take a standard Cobb-Douglas production function as an example:

$$Y = AK^{\alpha}L^{1-\alpha} \tag{5}$$

which, following the standard practice in the literature, we write in intensive form as

$$y = Ak^{\alpha} \tag{6}$$

where $k \equiv K/L$ and $y \equiv Y/L$.

Input markets are all perfectly competitive, where all agents have identical levels of productivity, and therefore, earn identical rates of competitive wages. Rents, on the other hand, accrue only to the optimal households or the 'capitalists'. From the profit maximization of the firm in perfect competition, therefore, we get the following expression for the wage rate (w) and the rental rate (r) in the economy:

$$w = A(1 - \alpha) k^{\alpha} \tag{7}$$
$$r = \frac{A\alpha}{k^{1-\alpha}} - \delta \tag{8}$$

where δ is the rate of depreciation of capital stock.

3.3 Poor hand-to-mouth household's consumption

Poor hand-to-mouth households consume whatever they earn from their wage income. Consumption by the 'hand-to-mouth' consumers, therefore, might be given by

$C^h = wL^h$. Dividing by L, and defining $c^h \equiv C^h/L$, we have

$$c^h = \lambda w \tag{9}$$

Substituting from (7) into (9):

$$c^h = \lambda A(1-\alpha)k^\alpha \tag{10}$$

We should, however, exercise caution by pointing out that c^h does not represent the per capita consumption by poor households. The latter might be calculated as

$$\begin{aligned} \frac{c^h}{L^h} &= w \\ &= A(1-\alpha)k^\alpha \end{aligned}$$

We note that the per capita consumption by the poor hand-to-mouth household depends on the stock of capital accumulated by the rich capitalist households.

3.4 Rich capitalist household's optimization problem

Optimal (rich, capitalist) consumers spend on consumption and investment in physical capital, out of the income received from wages and rent on capital. Hence, it follows that their budget constraint is given by

$$C^o + \dot{K} = wL^o + rK \tag{11}$$

where C^o is the consumption by the rich. Dividing (11) by L, and defining $c^o \equiv C^o/L$ and $k \equiv K/L$, we have the following expression for the budget constraint of optimal consumers:

$$\dot{k} = w(1-\lambda) + (r-n)k - c^o \tag{12}$$

We note that the total consumption, $C = C^h + C^o$, or in per capita terms, $c = c^h + c^o$. Substituting for the values of w and r from (7) and (8) respectively, into (12), we get

$$\dot{k} = A[1-\lambda(1-\alpha)]k^\alpha - (\delta+n)k - c^o \tag{13}$$

Let the rich capitalist households choose their consumption path by maximizing a CRRA utility function, given by

$$u(c^o) = \begin{cases} \dfrac{c^{o1-\sigma}-1}{1-\sigma} & \text{if } \sigma \neq 1 \\ \ln c^o & \text{if } \sigma = 1 \end{cases} \tag{14}$$

where σ is the elasticity of intertemporal substitution. We note that $u'(c^o) = 1/c^{o\sigma} > 0$ and $u'' = -\sigma/c^{o1+\sigma} < 0$.

Let us restrict ourselves to the case where $\sigma \neq 1$. Now, the optimization program for rich capitalist households is as follows:

$$\max_{c^o} \int_0^\infty \frac{c^{o1-\sigma} - 1}{1 - \sigma} e^{-\rho t} dt$$

$$\text{such that } \begin{cases} \dot{k} = A\left[1 - \lambda(1 - \alpha)\right] k^\alpha - (\delta + n) k - c^o \\ k(0) = k_0 > 0 \end{cases}$$

The current-valued Hamiltonian for the above problem would be given by

$$H = \frac{c^{o1-\sigma} - 1}{1 - \sigma} + \mu\left[A\{1 - \lambda(1 - \alpha)\} k^\alpha - (\delta + n) k - c^o\right] \tag{15}$$

where μ is the co-state variable, or the marginal imputed value (shadow price) of the per capita capital stock (Hoy, Livernois, McKenna, Rees, and Stengos 2011, page 1074). The first-order condition would be given by

$$\frac{\partial H}{\partial c^0} = \frac{1}{c^{o\sigma}} - \mu = 0 \tag{16}$$

$$\dot{\mu} = \rho\mu - \frac{\partial H}{\partial k} = \mu\left[\rho - \frac{\alpha A\{1 - \lambda(1 - \alpha)\}}{k^{1-\alpha}} + \delta + n\right] \tag{17}$$

Differentiating (16) w.r.t. t and substituting into (43), we get

$$\dot{c}^o = \frac{1}{\sigma}\left[\frac{\alpha A\{1 - \lambda(1 - \alpha)\}}{k^{1-\alpha}} - (\rho + \delta + n)\right] c^o \tag{18}$$

Equation (18) represents the Euler equation for consumption of rich capitalist households. In addition, from standard arguments, we have the transversality condition to hold, i.e.

$$\lim_{t \to \infty} e^{-\rho t} \frac{k}{c^{o\sigma}} = 0 \tag{19}$$

Next, we turn to the dynamics of social inequality.

3.5 Dynamics of inequality and social mobility

We now turn to λ, representing the fraction of households which are poor hand-to-mouth. Next, we depart from the conventional two-agent setup of Bilbiie (2008) and Gomes (2021) to introduce social mobility between the two social classes.

We recall that equation (2) represents the crucial condition which a hand-to-mouth household must satisfy to turn to a capitalist household. We now examine mobility between the two social classes.

First consider the set of hand-to-mouth households. We recall from (3) that $c_{\min,i} \sim U(0, \tilde{c})$. It follows that in any infinitesimally small period of time δt, given w, the probability of a hand-to-mouth household i escaping poverty and turning to a capitalist household by satisfying (2) is given by

$$\text{Prob}\left(w \geq c_{\min,i}\right) = \frac{w}{\tilde{c}} \tag{20}$$

Therefore, the mean fraction of hand-to-mouth households which makes a transition from hand-to-mouth to optimal capitalists in any infinitesimally small period of time, δt is given by $w/\tilde{c}\delta t$. Assuming that the economy is large enough so that we can ignore the deviation from the mean (Al-Najjar 2004; Sun 2006), we have the following expression of the rate of transition, ψ, of hand-to-mouth household to capitalist households:

$$\psi = \theta w \tag{21}$$

where $\theta \equiv 1/\tilde{c}$.

Once a household turns into a capitalist household, it has a combination of wage and rental income available to meet its expenditure, and hence is no longer constrained by (2). So it behaves like a standard Ramsey household, running intertemporal optimization to smoothen consumption as described in section 3.4. However, some of the capitalist households might face adverse idiosyncratic shocks, affecting their ability to continue as capitalists. We formalize this idea as follows. We assume that during every infinitesimally small period δt, adverse stochastic idiosyncratic shocks arrive through Poisson process on capitalist households with exogeneous probability $\phi\delta t$, pushing them to the set of poor hand-to-mouth households.[4] The evolution of mean proportion of hand-to-mouth households in the economy might now be expressed as follows:

$$\dot{\lambda} = \phi(1 - \lambda) - \theta\lambda w \tag{22}$$

Once again, we assume that the economy is large enough so that deviations from the mean can be ignored. Substituting from (7) into (22), we have

$$\dot{\lambda} = \phi(1 - \lambda) - A\theta(1 - \alpha)\lambda k^{\alpha} \tag{23}$$

Further, we note from (23) that the existing restrictions on parameters are sufficient to ensure that

$$\lim_{\lambda \to 0} \dot{\lambda} \geq 0 \tag{24}$$

[4]This transition might be preceded by periods in which they run down their individual holding of capital stock through consumption to zero. Alternately, an idiosyncratic shock might force them to declare bankruptcy and their holding of capital stock is divided among other capitalist households.

$$\lim_{\lambda \to 1} \dot{\lambda} \leq 0 \tag{25}$$

so it follows that for any initial condition $\lambda(0) \in (0, 1)$, we have

$$\lambda(t) \in (0, 1) \ \forall \, t \tag{26}$$

The dynamics of inequality represented by (23) makes it a predetermined state variable like the physical capital stock. Any change in λ through social mobility in either direction is an increment on its existing level. In other words, it is not a forward-looking jump variable which depends on expectations about future. This representation of λ is a modelling choice; however, it is largely in line with substantial persistence in the data on inequality that we observed in section 2.

3.6 Dynamical system

Equations (13), (18), and (23) might be considered together to constitute the dynamical system, representing the dynamic interaction between the per capita capital stock and consumption by the rich capitalist consumers in the presence of the dynamics of social interaction. We provide the complete dynamical system below for the sake of clarity.

$$\dot{k} = A\left[1 - \lambda(1-\alpha)\right] k^{\alpha} - (\delta + n)\, k - c^{o} \tag{13}$$

$$\dot{c}^{o} = \frac{1}{\sigma}\left[\frac{\alpha A\{1 - \lambda(1-\alpha)\}}{k^{1-\alpha}} - (\rho + \delta + n)\right] c^{o} \tag{18}$$

$$\dot{\lambda} = \phi(1-\lambda) - A\theta(1-\alpha)\lambda k^{\alpha} \tag{23}$$

with $(k, c^{o}, \lambda) \in \textbf{int}\, \mathbb{R}^{3+}$. The strict inequality, $k > 0$ and $c^{o} > 0$ is required for the steady state to be economically meaningful, whereas $\lambda > 0$ is required so that our model does not converge to the standard Ramsey-Cass-Koopmans neoclassical growth model. We should note here that out of the three variables in the above system, k and λ are the two state variables, whereas c^{0} is a control variable which depends on the decision taken by the rich 'capitalist' households.

It might be pointed out here that the above dynamical system represented by (13), (18) and (23), without equation (23) would resemble the model by Gomes (2021). The dynamical properties of the model without the distribution dynamics, as Gomes (2021) notes, is similar to that of the standard Ramsey-Cass-Koopmans neoclassical growth model.

3.7 Steady states

We can solve for the analytical steady states of the above dynamical system recursively. Since we are looking for economically meaningful solutions which lie in the interior of the positive orthant, we ignore trivial steady states. Let the non-trivial steady state(s) be represented by $\left(\bar{k}, \bar{c}^{o}, \bar{\lambda}\right)$. By setting the restriction $\bar{c}^{o} \neq 0$, we first solve for k and λ from (18) and (23), and then solve for c^{o} by plugging the steady states into (13). Following is a brief outline of this method. From $\dot{c}^{o} = 0$, we have

$$\frac{\alpha A\left\{1 - \bar{\lambda}(1-\alpha)\right\}}{\bar{k}^{1-\alpha}} = \rho + \delta + n$$

$$\Rightarrow \bar{\lambda} = \frac{1}{1-\alpha} - \frac{\rho + \delta + n}{A\alpha(1-\alpha)}\bar{k}^{1-\alpha} \tag{27}$$

$$\Rightarrow \left.\frac{\partial \bar{\lambda}}{\partial \bar{k}}\right|_{\dot{c}^{o}=0} = -\frac{\rho + \delta + n}{A\alpha \bar{k}^{\alpha}} < 0 \quad \forall\, \bar{k} > 0 \tag{28}$$

We note that an increase in the elasticity of substitution, α would shift the isocline upwards. Similarly, a rise the discount rate by the optimal households. or an increase in the rate of growth of population will make the isocline steeper.

Similarly, from $\dot{\lambda} = 0$, we have

$$\frac{1 - \bar{\lambda}}{\bar{\lambda}} = \frac{A\theta(1-\alpha)}{\phi}\bar{k}^{\alpha}$$

$$\Rightarrow \bar{\lambda} = \frac{\phi}{\phi + A\theta(1-\alpha)\bar{k}^{\alpha}} \tag{29}$$

$$\Rightarrow \left.\frac{\partial \bar{\lambda}}{\partial \bar{k}}\right|_{\dot{\lambda}=0} = -\frac{A\phi\theta\alpha(1-\alpha)}{\bar{k}^{1-\alpha}\left[\phi + A\theta(1-\alpha)\bar{k}^{\alpha}\right]^{2}} < 0 \quad \forall\, \bar{k} > 0 \tag{30}$$

We note that the isocline will get steeper with an increase in A (the parameter representing technological progress), ϕ (the rate at which adverse idiosyncratic shocks arrive at the optimal households, turning them into hand-to-mouth) or a fall in θ (the rate at which a higher wage rate enables hand-to-mouth households to turn into optimal households).

In other words, both $\dot{c}^{o} = 0$ and $\dot{\lambda} = 0$ are negatively sloped curves in $k - \lambda$ space, where the steady state values of k and λ are determined by the intersection of these two curves. We can solve for the steady state values, $\bar{k}$ and $\bar{\lambda}$ from equations (27) and (29). The steady state value, $\bar{k}$ would be given by solution to the following polynomial equation:

$$A^{2}\theta\alpha(1-\alpha)k^{\alpha} - \phi(\rho + \delta + n)k^{1-\alpha} - A\theta(1-\alpha)(\rho + \delta + n)k + A\phi\alpha^{2} = 0 \tag{31}$$

Once we determine the value of $\bar{k}$ from (31), we can compute the value of $\bar{\lambda}$ from either either (27) or (29).

However, it is difficult to analytically compute the roots of the polynomial of the form given in (31). So, we use a numerical example to demonstrate the existence of at least one economically meaningful non-trivial steady state. We perform a numerical analysis of the dynamical system with the following parameter configurations:

$$A = 1; \quad \alpha = 0.3; \quad \rho = 0.02; \quad n = 0.01; \quad \sigma = 0.2; \quad \delta = 0.08; \quad \theta = 0.1; \quad \phi = 0.1$$

Equations (27) and (29) for the above numerical example is shown below in figure 4.

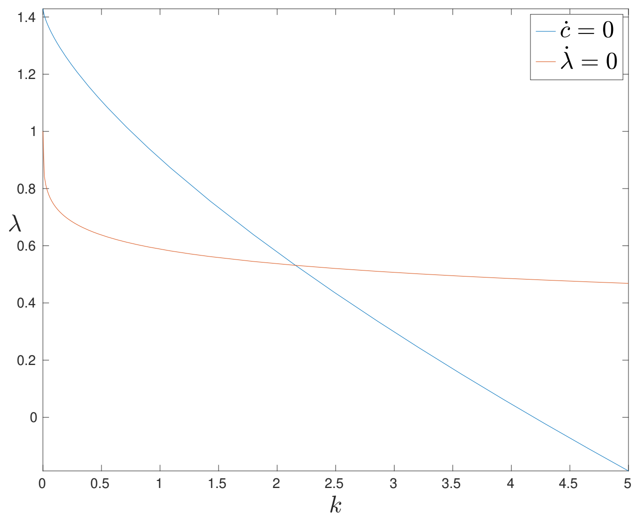


Figure 4: Isoclines $\dot{c} = 0$ and $\dot{\lambda} = 0$ in $k - \lambda$ space

We can see from figure 4 that there exists at least one non-trivial point of intersection. This provides us with the steady state values, $\bar{k}$ and $\bar{\lambda}$, which for our numerical example can be calculated to be $\bar{k} = 2.1568$ and $\bar{\lambda} = 0.5315$.

Finally, we can substitute for the steady state values of $\bar{k}$ and $\bar{\lambda}$ into (13) to compute the steady state value of $\bar{c}^o$.

$$\bar{c}^o = A\left[1 - \bar{\lambda}(1 - \alpha)\right]\bar{k}^{\alpha} - (\delta + n)\bar{k} \tag{32}$$

Figure 5 below shows the surf plot of c^o as a function of k and λ along $\dot{k} = 0$ for the numerical example described above. We note that

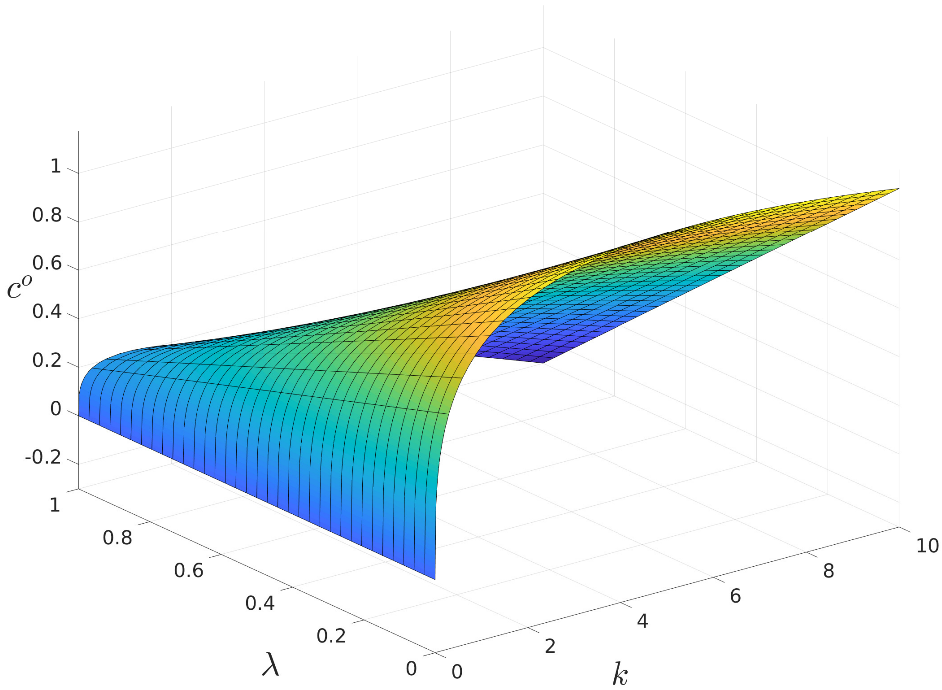


Figure 5: c^o as a function of k and λ along $\dot{k} = 0$

$$\left.\frac{\partial \bar{c}^o}{\partial \bar{k}}\right|_{\dot{k}=0} = \frac{\alpha A\{1 - \lambda(1-\alpha)\}}{k^{1-\alpha}} - (\delta + n)$$

$$\Rightarrow \left.\frac{\partial \bar{c}^o}{\partial \bar{k}}\right|_{\dot{k}=0} = \rho > 0 \qquad \text{[from (18) at the steady state]} \qquad (33)$$

In other words, the steady state per capita consumption by the capitalists depends directly on the steady state per capita capital stock. Of course, this is on expected lines, as a higher steady state per capita capital stock leads to a higher rental income as well as higher wage income for the capitalists, leading to a higher income available for both consumption and savings. Further,

$$\left.\frac{\partial \bar{c}^o}{\partial \bar{\lambda}}\right|_{\dot{k}=0} = -A(1-\alpha)k^{\alpha} < 0 \quad \forall \bar{k} > 0 \qquad (34)$$

i.e. along the isocline $\dot{k} = 0$, the steady state rate of per capita consumption by the capitalists depend inversely on the proportion of hand-to-mouth consumers. In other words, a higher inequality adversely affects the consumption of not only the hand-to-mouth households but also the optimal capitalist households. This is because of the inability of the hand-to-mouth households to save based on an intertemporal optimization exercise, leading to lower than efficient output available for consumption. We should also note that

$$\left.\frac{\partial \bar{c}^o}{\partial \bar{k}}\right|_{\bar{k}=0} > 0 \quad \forall\, \bar{\lambda} < \frac{1}{1-\alpha} - \frac{\delta + n}{A\alpha\,(1-\alpha)}\bar{k}^{1-\alpha}$$

In other words, a higher per capita capital stock increases the consumption for the optimal households only below a threshold level of inequality. Above this, any further increase in capital accumulation will not translate to higher consumption for optimal households in the steady state.

We can determine the steady state by substituting the equilibrium values of k and λ into (13). For our numerical example, this non-trivial steady state is represented by

$$\left(\bar{k}, \bar{c}^0, \bar{\lambda}\right) = (2.1568, 0.5967, 0.5315) \tag{35}$$

with the per capita output at 1.2593.[5]

It would be evident from the above discussion that under certain conditions, we will get at least one economically meaningful steady state such that $(k, c^o, \lambda) \in \textbf{int}\, \mathbb{R}^{3+}$. We further note, from (32), that for $\lambda = 0$ (i.e. all households are optimal capitalists), the relationship between the steady state values of k and c would resemble a standard Ramsey-Cass-Koopmans neoclassical growth model.

We further note, from a perturbation of the parameters ϕ and θ that

$$\frac{d\bar{k}}{d\phi} < 0;\ \frac{d\bar{\lambda}}{d\phi} > 0;\ \frac{d\bar{k}}{d\theta} > 0;\ \frac{d\bar{\lambda}}{d\theta} < 0 \tag{36}$$

In other words, a higher rate of social mobility from capitalist to hand-to-mouth households will have a negative impact on steady state rate of capital accumulation and a positive impact on the steady state proportion of population who are hand-to-mouth. Similarly, a higher rate of social mobility from hand-to-mouth to capitalist households will have a positive impact on steady state rate of capital accumulation and a negative impact on the steady state proportion of population who are hand-to-mouth.

[5] We should note here that given the high degree of nonlinearity, there are 21 steady states. However, many of them are either trivial or contain imaginary component. There is only one steady state which lies within the real positive orthant, and hence, is economically meaningful.

3.8 Local stability

The jacobian matrix of the dynamical system represented by (13), (18), and (23) evaluated at the non-trivial steady state is as follows:

$$J|_{(\bar{k},\bar{c}^o,\bar{\lambda})} = \begin{bmatrix} \alpha A\rho - (1 - A\alpha)(\delta + n) & -1 & -A(1-\alpha)\bar{k}^{\alpha} \\ -\frac{A}{\sigma}\alpha(1-\alpha)\left\{1 - \bar{\lambda}(1-\alpha)\right\}\frac{\bar{c}^o}{\bar{k}^{2-\alpha}} & 0 & -\frac{A\alpha(1-\alpha)\bar{c}^o}{\sigma\bar{k}^{1-\alpha}} \\ -\frac{A\theta\alpha(1-\alpha)\bar{\lambda}}{\bar{k}^{1-\alpha}} & 0 & -\phi - A\theta(1-\alpha)\bar{k}^{\alpha} \end{bmatrix} \tag{37}$$

The general form characteristic equation to the above jacobian is given by

$$\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0 \tag{38}$$

where

$$a_1 = \delta + n + \phi + A\theta\bar{k}^{\alpha}(1-\alpha) - A\alpha(\rho + \delta + n) \tag{39}$$

$$a_3 = -\frac{A\alpha(1-\alpha)\left[A\theta\bar{k}^{\alpha}(1-\alpha)\left(1-\bar{\lambda}\right) + \phi\left\{1 - \bar{\lambda}(1-\alpha)\right\}\right]\bar{c}^o}{\sigma\bar{k}^{2-\alpha}} \tag{40}$$

We immediately note that existing restrictions on the parameters are sufficient to ensure that $a_3 < 0$, i.e. product of the three characteristic roots is positive. In other words, *at least one* of the three characteristic roots must be positive; the other two roots must carry the same sign – either both positive or both negative. Hence, we can conclude that the economically meaningful steady state(s) is (are) either a focus or a saddle. We further note that the Blanchard-Kahn condition is satisfied, since there is one control variable, c^o which requires at least one eigenvalue to be positive.

Given that the dynamical system represented by (13), (18) and (23) is highly non-linear and parameterized, it is difficult to analyze it any further analytically. Hence, we perform a numerical analysis of the dynamical system with the parameter configurations provided in section 3.7 for the steady state described in (35). By linearizing the dynamical system around this economically meaningful steady state, we find that the real parts of the characteristic roots to the jacobian matrix at this point is given by

$$(0.3352, -0.3549, -0.1484)$$

In other words, we can conclude that the economically meaningful steady state is a saddle-point, where a two-dimensional stable manifold intersects with a one-dimensional unstable manifold. This ensures that the Blanchard-Kahn condition is satisfied.

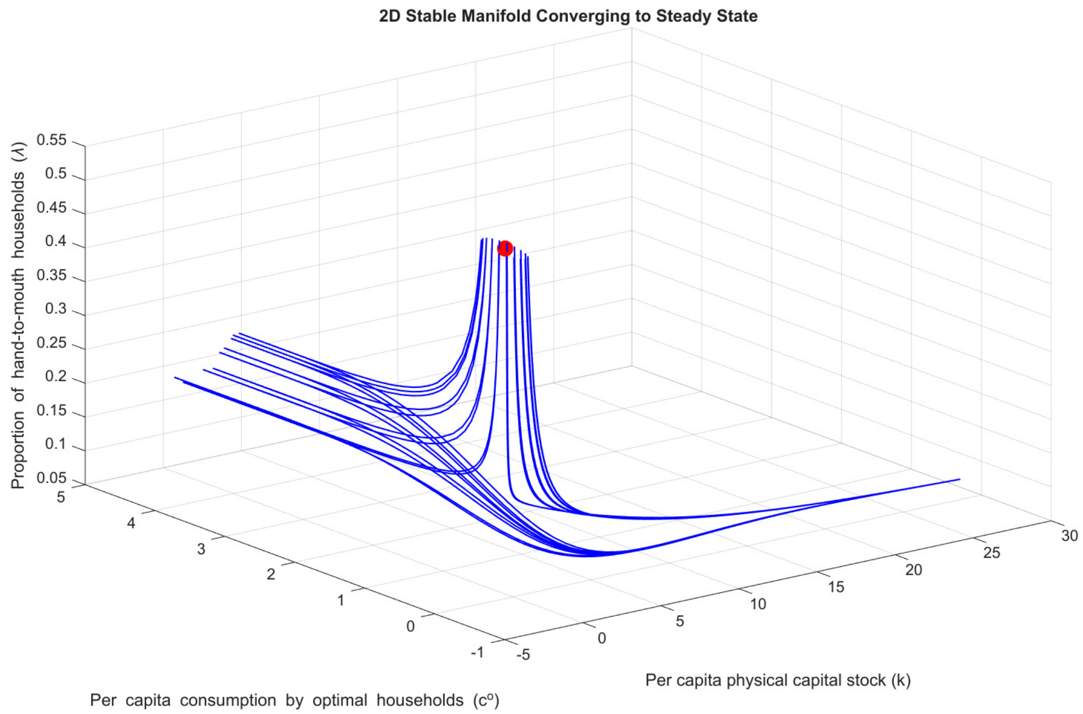


Figure 6: Saddle-path trajectories

A saddle-point stability of the economically meaningful steady state is consistent with the literature on neoclassical growth models; for instance, the textbook Ramsey-Cass-Koopmans model, where the agents are homogeneous and representative, also shows this type of stability property (Bénassy 2011). The trajectories around this type of steady state are generally sensitive to initial conditions. In our model, trajectories starting from the stable manifold will converge to the economically meaningful steady state, whereas any other initial point will either converge to a trivial steady state (where $\bar{c}^o = 0$) or explode. Generally the literature on neoclassical growth models argue that the optimal agents choose trajectories starting from the stable manifold in order to satisfy the transversality condition. We stick to this interpretation for our model. The saddle-point stability is shown below.

4 Inefficiency of the competitive markets under inequality

Next, we consider the social planner's problem. Unlike the households under decentralized market economy, the social planner's problem would consist of maximizing the utility of all households (including the hand-to-mouth households). This would reduce to the standard textbook version of the neoclassical growth model. The social planner maximizes the utility of representative household, without making a distinction between the rich and the poor, in the presence of the resource constraint. This leads us to the following optimization problem:

$$\max_{c} \int_{0}^{\infty} \frac{c^{1-\sigma}-1}{1-\sigma} e^{-\rho t} dt$$

$$\text{such that } \begin{cases} \dot{k} = Ak^{\alpha} - (\delta + n)\,k - c \\ k(0) = k_0 > 0 \end{cases}$$

The current-valued Hamiltonian for the above problem would be given by

$$H = \frac{c^{1-\sigma}-1}{1-\sigma} + \mu\left[Ak^{\alpha} - (\delta + n)\,k - c\right] \tag{41}$$

The first-order condition would be given by

$$\frac{\partial H}{\partial c} = \frac{1}{c^{\sigma}} - \mu = 0 \tag{42}$$

$$\dot{\mu} = \rho\mu - \frac{\partial H}{\partial k} = \mu\left[\rho - \frac{\alpha A}{k^{1-\alpha}} + \delta + n\right] \tag{43}$$

Differentiating (16) w.r.t. t and substituting into (43), we get

$$\dot{c} = \frac{1}{\sigma}\left[\frac{\alpha A}{k^{1-\alpha}} - (\rho + \delta + n)\right] c \tag{44}$$

Equation (44) represents the Euler equation for the social planner. In addition, from standard arguments, we have the transversality condition to hold, i.e.

$$\lim_{t \to \infty} e^{-\rho t} \frac{k}{c^{\sigma}} = 0 \tag{45}$$

Note that, unlike the rich households, social planner's problem requires the consumption of all the social classes to grow at the same rate.

Now, from (27), in a decentralized competitive market economy with inequality, in the steady state, for any $\bar{\lambda}$, we have

$$\bar{k}\Big|_{DCM} = \left[\frac{\alpha A\left\{1 - \bar{\lambda}\,(1-\alpha)\right\}}{\rho + \delta + n}\right]^{\frac{1}{1-\alpha}} \tag{46}$$

On the other hand, the per capita capital stock in steady state for the social planner's problem is given by

$$\bar{k}\Big|_{SP} = \left[\frac{\alpha A}{\rho + \delta + n}\right]^{\frac{1}{1-\alpha}} \tag{47}$$

We recall that $\lambda \in (0, 1)$. Hence, it follows that $\bar{k}\big|_{SP} > \bar{k}\big|_{DCM}$.

As an illustration, we consider the numerical example we discussed in section 3.7. The social planner's problem in this case would yield a steady state per capita capital stock of 4.1925 and a per capita output of 1.5372, which are both higher than the steady state for the decentralized market equilibrium.

In other words, we can conclude that in general, in the presence of inequality, the competitive market equilibrium will be inefficient. Typically, the capital accumulation will be at suboptimal level, leading to suboptimal levels of output. The inefficiency of the decentralized market economy stems mainly from the presence of hand-to-mouth consumers, who do not optimize.

We need to, however, be careful in interpreting the representation of the social planner's problem above. The social planner's problem above assumes a world without inequality where all consumers save. In case the planner actually wants to implement the efficient solution, it needs to figure out how to generate the savings for the higher levels of capital accumulation associated with the efficient solution. In a closed economy setup of our model, this might either come from some sort of forced savings. In this case, how the distribution of this additional forced savings across various social classes will have welfare implications. We turn to some of these concerns below.

5 Efficiency of capital income tax

There has been a substantial discussion in the literature comparing the welfare implications of a tax on capital income versus a tax on capital. In the traditional neoclassical model, these two types of taxes are equivalent. However, more recently, Guvenen, Kambourov, Kuruscu, Ocampo, and Chen (2023) argued that these two types of taxes will have difference welfare implications in the presence of heterogeneous returns on capital, since the capital income tax will tend to reward inefficiency by taxing the more productive capitalists at a higher rate, whereas a wealth tax will affect everyone equally by taxing every capitalist at the same rate. While our model does include inequality, it does not take into consieration differences in return to capital. However, in our model, if the tax revenues are used to provide income support to the poor, we also need to take into account the welfare implications of the social mobility induced by income redistribution. We examine this in the following paragraphs below.

We first consider the effect of these taxes on the budget constraint of the rich capitalist household. Let there be a tax at the rate τ imposed on rental income. This leads to a modification to the budget constraint set up in (11) as follows:

$$C^o + \dot{K} = wL^o + r(1-\tau)K \tag{48}$$

This leads to a modification to (13) as follows:

$$\dot{k} = A[1-\lambda(1-\alpha)-\alpha\tau]k^\alpha - (\delta(1-\tau)+n)k - c^o \tag{49}$$

The current-valued Hamiltonian representing the representative capitalist household's optimization problem in equation (15) is now modified as follows:

$$H = \frac{c^{o\,1-\sigma}-1}{1-\sigma} + \mu\{A[1-\lambda(1-\alpha)-\alpha\tau]k^\alpha - [\delta(1-\tau)+n]k - c^o\} \tag{50}$$

This leads to the following modification to the Euler Equation (18) as follows.

$$\dot{c}^o = \frac{1}{\sigma}\left\{\frac{\alpha A[1-\lambda(1-\alpha)-\alpha\tau]}{k^{1-\alpha}} - [\rho+\delta(1-\tau)+n]\right\}c^o \tag{51}$$

We assume that the entire revenue from taxing the capital income of the rich capitalists is distributed by transfers among poor households. Given that the total tax revenue is τrK, each household gets a share given by $\tau rK/L^H = \tau rk/\lambda$. This additional transfer becomes available to the poor households for consumption, and offers them a somewhat higher probability of escaping poverty. This leads to a modification to (21) as follows:

$$\psi = \theta\left(w + \frac{\tau rk}{\lambda}\right) \tag{52}$$

Hence, evolution of mean proportion of hand-to-mouth households in the economy represented by (22) also now needs to be modified as follows:

$$\dot{\lambda} = \phi(1-\lambda) - \theta\lambda\left(w + \frac{\tau rk}{\lambda}\right) \tag{53}$$

The final differential equation for evolution of λ is now a modified version of (23) as follows:

$$\dot{\lambda} = \phi(1-\lambda) - A\theta(1-\alpha)\lambda k^\alpha - \theta\tau(A\alpha k^\alpha - \delta k) \tag{54}$$

We note from (54) that the existing restrictions on parameters are sufficient to ensure that $\lim_{\lambda\to 1}\dot{\lambda} \leq 0$. However, unlike the original model, we need to put additional restrictions on τ to ensure that $\lim_{\lambda\to 0}\dot{\lambda} \geq 0$ so it follows that for any initial condition $\lambda(0) \in (0,1)$, we have $\lambda(t) \in (0,1)\ \forall\ t$. This additional restriction looks reasonable, since the main objective of capital taxation is to address poverty. If λ is very low, then it is no longer rational from the point of view of the policymaker to pursue such an income redistribution measure anymore.

The system of differential equations represented by (49), (51) and (54) now constitutes our new dynamical system in the presence of capital taxation and redistribution through transfers among poor households.

The steady states for the modified dynamical system might be examined in the same way as the original system. From $\dot{c}^o = 0$, with $\bar{c}^o \neq 0$ we have a modified version of (27):

$$\bar{\lambda} = \frac{1 - \tau\alpha}{1 - \alpha} - \frac{\rho + \delta(1 - \tau) + n}{A\alpha(1 - \alpha)}\bar{k}^{1-\alpha} \tag{55}$$

On comparing (55) with the isocline represented by (27), we find that the capital income tax leads to a reduction in both the intercept and the slope (i.e. makes the isocline flatter to the origin). In other words, there is a downward shift of the isocline. Similarly, from $\dot{\lambda} = 0$, we have the modified version of (29):

$$\bar{\lambda} = \frac{\phi - \tau\theta(A\alpha k^{\alpha} - \delta k)}{\phi + A\theta(1 - \alpha)\bar{k}^{\alpha}} \tag{56}$$

It would be evident from (56) that as long as

$$\bar{k} \in \left(0, \left(\frac{A\alpha}{\delta}\right)^{1/(1-\alpha)}\right)$$

it follows that right hand side of (56) is smaller than the right hand side of (29), i.e.

$$\frac{\phi - \tau\theta(A\alpha k^{\alpha} - \delta k)}{\phi + A\theta(1 - \alpha)\bar{k}^{\alpha}} < \frac{\phi}{\phi + A\theta(1 - \alpha)\bar{k}^{\alpha}} \tag{57}$$

In other words, imposition of a capital income tax leads to a downward shift of the $\dot{\lambda} = 0$ isocline.

To sum up, we note that imposition of a tax on capital income will shift both the isoclines in figure 4 downwards.

The overall effect of an imposition of tax on capital income on steady state level of per capita consumption by the capitalists and the proportion of hand-to-mouth households is ambiguous. We consider a case where both the isoclines are downward-sloping, and $\dot{c}^o = 0$ has a higher slope than $\dot{\lambda} = 0$ at the point of intersection. In this case, there are two channels which will work in opposite direction:

1. A downward shift of $\dot{c}^o = 0$ isocline will tend to reduce k and increase λ. The transmission of this process will work through the capitalist household's optimization

exercise. A tax on rental income will reduce the disposable income of the capitalist household, leaving less income available for both consumption and savings. This will slow down the rate of capital accumulation, lowering the wage income as well. A lower wage income will also push more households into poverty, leading to an increase in λ.

2. On the other hand, a downward shift in the $\dot{\lambda} = 0$ isocline will lead to an increase in k and a fall in λ. A fall in λ will be the direct outcome of an income redistribution, which will pull up some poor households to turn them into capitalist households. The transition of these households into capitalist households will also lead to an increase in capital accumulation, leading to an increase in both wage and rental income, eventually improving the consumption of both types of households.

We should further note that imposition of a capital income tax also makes the $\dot{\lambda} = 0$ isocline flatter, strengthening the channel through which the capital income taxation improves capital accumulation and reduces inequality, relative the channel which works in the opposite direction.

6 Historically disadvantaged groups: The 'resilient' hand-to-mouth

In the previous sections, we considered a model of social mobility where in each period, a certain fraction of hand-to-mouth households are randomly selected to make a transition to the set of rich capitalist households. The existing literature on heuristic behavior indicates, however, the presence of resilient agents who are unaffected by ups and downs in the rest of the economy (see, for instance, Cifarelli and Paladino 2018). In the context of our model, the existing literature points to the case of historically disadvantaged groups based on identities like race (see, for instance, Wolff 2012; Chelwa, Maboshe, and and 2024) or caste (Zacharias and Vakulabharanam 2011; Tagade and Thorat 2020; Tiwari, Goli, Siddiqui, and Salve 2022). In other words, historically disadvantaged groups are often completely excluded from the financial markets, irrespective of the aggregate economic dynamics.

The above insights lead us to suggest the following modification to our model. Let $\underline{\lambda}$ be the proportion of agents who are resilient hand-to-mouth. In other words, these agents, because of belonging to certain historically disadvantaged categories, are perpetually deprived from holding wealth even when when the economy is doing well. Therefore, only a fraction $\lambda - \underline{\lambda}$ are available for selection in the transition to rich capitalist households. This leads us to modify equation (22) as follows:

$$\dot{\lambda} = \phi(1-\lambda) - \theta\left(\lambda - \underline{\lambda}\right)w \tag{58}$$

Equation (23), therefore, is modified as follows:

$$\dot{\lambda} = \phi(1-\lambda) - A\theta(1-\alpha)\left(\lambda - \underline{\lambda}\right)k^{\alpha} \tag{59}$$

We note from (59) that the existing restrictions on parameters are now sufficient to ensure that

$$\lim_{\lambda\to\underline{\lambda}} \dot{\lambda} \geq 0 \tag{60}$$

$$\lim_{\lambda\to 1} \dot{\lambda} \leq 0 \tag{61}$$

so it follows that for any initial condition $\lambda(0) \in \left(\underline{\lambda}, 1\right)$, we have

$$\lambda(t) \in \left(\underline{\lambda}, 1\right)\ \forall\, t \tag{62}$$

Equation (59) now replaces (23) in the new modified dynamical system, along with equations (13), (18).

Solving for $\dot{\lambda} = 0$ from the modified differential equation represented by (59), we get the new isocline as follows:

$$\bar{\lambda}_{\text{new}} = \underline{\lambda} + \frac{\left(1-\underline{\lambda}\right)\phi}{\phi + A\theta(1-\alpha)\bar{k}^{\alpha}} \tag{63}$$

We can rewrite (63) as follows:

$$\bar{\lambda}_{\text{new}} = \underline{\lambda}\left[1 - \frac{\phi}{\phi + A\theta(1-\alpha)\bar{k}^{\alpha}}\right] + \frac{\phi}{\phi + A\theta(1-\alpha)\bar{k}^{\alpha}} \tag{64}$$

On comparing the right-hand-side of (64) with that of (29), we find that

$$\bar{\lambda}_{\text{new}} \geq \bar{\lambda} \quad \forall\, \bar{k} > 0 \tag{65}$$

In other words, as expected, presence of a historically disadvantaged group of resilient hand-to-mouth poor consumers lead to a greater fraction of poor agents along the equilibrium growth path for any given trajectory of per capita stock of physical capital. We further note the following after partial differentiation of (64):

$$\left.\frac{\partial\bar{\lambda}_{\text{new}}}{\partial\bar{k}}\right|_{\dot{\lambda}=0} = -\frac{A\phi\theta\alpha(1-\alpha)\left(1-\underline{\lambda}\right)}{\bar{k}^{1-\alpha}\left[\phi + A\theta(1-\alpha)\bar{k}^{\alpha}\right]^{2}} < 0 \quad \forall\, \bar{k} > 0 \tag{66}$$

We can see that

$$\left|\frac{\partial \bar{\lambda}_{\text{new}}}{\partial \bar{k}}\right|_{\dot{\lambda}=0} \leq \left|\frac{\partial \bar{\lambda}}{\partial \bar{k}}\right|_{\dot{\lambda}=0}$$

In other words, compared to (30) the isocline represented by (66) is flatter due to the presence of a historically disadvantaged group of resilient hand-to-mouth poor consumers. This is because the resilient hand-to-mouth consumers are unaffected by any shifts in equilibrium capital stock, since because of certain structural disadvantages they continue to remain poor. The new isocline also has a larger vertical intercept, representing a higher proportion of poor hand-to-mouth consumers in equilibrium.

From figure 4, it would be evident that a flatter and vertically upward shifted $\dot{\lambda} = 0$ isocline will result in a lower $\bar{k}$ and a higher $\bar{\lambda}$ in the steady state. Further, from (33) and (34), a lower $\bar{k}$ and a higher $\bar{\lambda}$ will result in a lower $\bar{c}^o$ in the steady state. In other words, we can conclude that the presence of resilient hand-to-mouth consumers will result in lower per capita capital stock (and per capita output), lower level of per capita consumption by the rich capitalist consumers, and a higher proportion of poor hand-to-mouth consumers. Further, a lower level of per capita physical capital stock also results in a lower consumption by the poor hand-to-mouth households.

For instance, for the numerical example discussed in section 3.7, if 30 percent of population is resilient hand-to-mouth, i.e. $\underline{\lambda} = 0.3$, then the non-trivial steady state will be given by

$$\left(\bar{k}_{\text{new}}, \bar{c}^0_{\text{new}}, \bar{\lambda}_{\text{new}}\right) = (1.6459, 0.4554, 0.6861)$$

and a per capita GDP of 1.1612. When we compare this with the earlier numerical steady state in section 3.7, we find that the presence of the resilent hand-to-mouth leads to a relatively lower level of per capita capital stock and per capita consumption by the capitalist households, and a higher proportion of poor hand-to-mouth households in the steady state. The per capita output is also lower in the steady state in the presence of resilient hand-to-mouth households.

A summary of the steady state per capita capital stock and the per capita output under three alternative setups is provided below in table 4.

It is clear from table 4 that the economy and the society pays a substantial cost of inefficiency due to inequality. This cost is even higher in case a certain fraction of population are resilient hand-to-mouth. This pertains to a situation where certain historically

Table 4: Steady state per capita capital stock and per capita output under alternate setups

	$\bar{k}$	$\bar{y}$
Social planner's problem	4.1925	1.5372
Decentralized market economy w/o resilient HTM	2.1568	1.2598
Decentralized market economy with resilient HTM	1.6459	1.1612

disadvantaged and dispossessed social groups are not in a position to benefit from social mobility even when the economic situation is favorable. This suggests that specific policy initiatives like affirmative action measures targeted towards such historically disadvantaged social groups are likely to benefit not only the members of these groups, but also others (including the rich capitalist households) due to the efficiency gains from these measures.

7 Conclusions

In the above sections, we formulated a neoclassical growth model with heterogeneous agents and endogenous distribution dynamics. The main results and contributions from this exercise can be summarized as follows:

1. We include a constraint for households to participate in capital accumulation, involving a heterogeneous minimum consumption floor which must be met by current wage. The resultant system is reduced to a two-agent model with limited asset market participation (LAMP) similar to the one proposed by Bilbiie (2008) and Gomes (2021).

2. Presence of endogenous inequality in a standard neoclassical growth model leads to three-dimensional dynamical system, where the steady state levels of per capita capital stock, per capita consumption by different social classes as well as the level of inequalities are jointly determined. More specifically, the Euler equation from optimization exercise by the optimal households and the inequality dynamics jointly determine the per capita capital stock and the level of inequality in the economy. The resource constraint in the economy then determines the steady state consumption by different social classes, given the steady state levels of per capita capital stock and inequality.

3. In the presence of endogenous inequality, the decentralized market equilibrium is inefficient and the first theorem of welfare economics does not hold. Various forms of redistribution measures to lift the hand-to-mouth households out of poverty into a participant in the process of capital accumulation improves welfare. More specifically, a tax on capital income may improve efficiency under certain conditions.

4. Welfare might also be improved by supporting optimal 'capitalists' at the time of economic distress, preventing them from turning into hand-to-mouth households.

5. Endogenous inequality affects the business cycle dynamics. This is because consumption smoothening varies over the stages of business cycles. During periods of recessions, there is a transition of certain households from optimal to hand-to-mouth type. Hand-to-mouth households respond more strongly to current income than optimal households. So consumption falls faster than current income during periods of recession, as an increase in number of poor during business downturn sharpens the relation between current income and consumption. On the other hand, during the upward phase of the business cycle, households shift from hand-to-mouth to optimal. Thus consumption smoothening increases, and consumption increases slower than income during upward phases of business cycles.

6. The presence of certain historically disadvantaged groups or communities might be a source of additional inefficiency in the economy. If these groups, due to their historical disadvantage or discrimination, are not able to make a transition out of poverty into the asset market then specific and targeted policy measures might be required. Affirmative action measures might be effective in these situations in improving welfare.

7. Policy implications: Given that the decentralized market equilibrium is inefficient, various types of policy regimes might be adopted to improve welfare, which includes taking over the control of the economy, various types of redistribution policies as well as policies to support consumption during periods of crisis.

Overall, we find that the distribution dynamics affect the process of capital accumulation and growth. Therefore, income distribution should be taken seriously, not just as an end but also because of its impact on the output trajectory of an economy.

Finally, we should point out that we do not attempt to develop a full-fledged heterogeneous agent model with heterogeneity in wages, capital stock and wealth holdings.

We concede that all these factors are important for a more complete analysis of the relationship between inequality and economic growth. There have already been a substantial literature in this direction. However, the focus of this study was on developing a simple tractable story of economic growth with mobility between social classes to capture the role played by inequality and poverty. We feel that the current study might be considered as a first step in this direction. We leave the incorporation of other types of heterogeneity in a tractable macroeconomic framework as possible future extensions of the current study.

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